Practice Exam #5

1. Find the vertex of the parabola and indicate whether it opens upward, downward, to the left, or to the right.
   a. $y = x^2 + 4x$
   b. $x = y^2 + 4y$
   c. $x = -3y^2 + 6y - 11$
   d. $y = -4x^2 - 5$

2. Identify whether each equation, when graphed, will be a parabola, circle, ellipse, or hyperbola.
   a. $x^2 - 4y^2 = 16$
   b. $4y - x^2 = 12$
   c. $x^2 + 4y^2 = 100$
   d. $25x^2 + 25y^2 = 100$

3. Graph each equation.
   a. $x^2 + y^2 = 100$
   b. $4(x-3)^2 + (y-2)^2 = 16$
   c. $\frac{y^2}{16} - \frac{x^2}{25} = 1$
   d. $(x+2)^2 - 9y^2 = 36$
   e. $x^2 + y^2 - 4x + 6y = 3$
   f. $\frac{(x-2)^2}{49} + \frac{y^2}{81} = 1$

4. Graph each inequality.
   a. $y^2 - 4y - x + 7 < 0$
   b. $(x+1)^2 + (y-2)^2 \le 9$
   c. $\frac{(x-2)^2}{36} + \frac{(y+4)^2}{25} > 1$

5. Graph the solution for the following systems.
   a. $\begin{cases} 25x^2 + 9y^2 < 225 \\ 4x^2 + y^2 \ge 16 \end{cases}$
   b. $\begin{cases} x^2 \le 1 - y \\ x^2 \le y + 1 \end{cases}$

6. Write an equation of a circle with the following specifications.
   a. Centered at the origin with radius 1.
   b. Centered at the origin with radius $\sqrt{7}$.
   c. Centered at $(-2,1)$ with radius 3.
   d. Centered at $(3,-7)$ with radius 11.

7. Solve the following systems. Write as an ordered pair.

(a) $\begin{cases} y = 3x - 5 \\ x^2 + y^2 = 5 \end{cases}$

(b) $\begin{cases} x^2 - 3y^2 = -1 \\ 2x^2 - 7y^2 = -5 \end{cases}$

(c) $\begin{cases} y = x^2 - 1 \\ y = -4x - 5 \end{cases}$

8. Find the first five terms and the 100th term of the sequence.

a. $a_n = 4 - n$
b. $b_n = 2^{n-1}$
c. $c_n = \frac{n^2}{n+1}$

9. Find the general term (formula) for the following sequences.

a. $1,2,3,4,\ldots$
b. $9,6,3,0,\ldots$
c. $8,4,2,1,\ldots$
d. $2,6,18,54,\ldots$
e. $5,11,21,35,53\ldots$

10. Use summation (sigma) to write the following series.

a. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}$
b. $2+7+12+17+22$
c. $\frac{1}{4}+\frac{2}{5}+\frac{1}{2}+\frac{4}{7}+\frac{5}{8}$

11. Evaluate the sum.

a. $\sum_{i=1}^{9} 2^i$

b. $\sum_{i=5}^{11} \frac{1}{i+3}$

Math 70 Practice Exam #5

* This practice exam is missing graphing parabolas!
* Also equation of axis of symmetry!
↳ done on p. 3-4 of these solutions.

① Find vertex, direction

a) $y = x^2 + 4x$

y and $x^2 \Rightarrow$ up or down — look at squared variable

$a=1$ in $1x^2 \Rightarrow \boxed{\text{up}}$ — is $a>0$ or $a<0$?

vertex $x = -\frac{b}{2a} = \frac{-4}{2(1)} = \frac{-4}{2} = -2$ — Vertex formula

$y = (-2)^2 + 4(-2)$ — subst back

$y = 4 - 8 = -4$ — simplify

$\boxed{\text{vertex } (-2,-4)}$ axis $x=-2$ — ordered pair

b) $x = y^2 + 4y$

x and $y^2 \Rightarrow$ left or right

$a=1$ in $1y^2 \Rightarrow \boxed{\text{right}}$

vertex $y = \frac{-b}{2a} = \frac{-4}{2(1)} = \frac{-4}{2} = -2$ — vertex formula finds y-coord when left or right!

$x = (-2)^2 + 4(-2)$

$= 4 - 8 = -4$

$\boxed{\text{vertex } (-4,-2)}$ axis $y=-2$

c) $x = -3y^2 + 6y - 11$

x and $y^2 \Rightarrow$ left or right

$a=-3$ in $-3y^2 \Rightarrow \boxed{\text{left}}$

Vertex $y = \frac{-b}{2a} = \frac{-6}{2(-3)} = \frac{-6}{-6} = 1$

$x = -3(1)^2 + 6(1) - 11$

$= -3 + 6 - 11$

$= -8$

$\boxed{\text{vertex } (-8,1)}$ axis $y=1$

d) $y = -4x^2 - 5$

y and $x^2 \Rightarrow$ up or down

$a = -4$ in $-4x^2 \Rightarrow \boxed{\text{down}}$

$$x = \frac{-b}{2a} = \frac{-0}{2(-4)} = 0$$

$y = -4(0^2) - 5 = -5$

vertex $\boxed{(0, -5)}$ axis $x = 0$

Also: $y = -4(x-0)^2 - 5$

means $y = -4(x-h)^2 + k$

vertex $\boxed{(0, -5)}$

② Identify shape

a) $x^2 - 4y^2 = 16$

x^2 and $y^2 \Rightarrow$ not a parabola

x^2 subtract $y^2 \Rightarrow \boxed{\text{hyperbola}}$

b) $4y - x^2 = 12$

y and $x^2 \Rightarrow \boxed{\text{parabola}}$

c) $x^2 + 4y^2 = 100$

x^2 and $y^2 \Rightarrow$ not a parabola

subtract $\Rightarrow$ not a hyperbola

coefficients $1x^2$ and $4y^2$ different $\Rightarrow$ not a circle

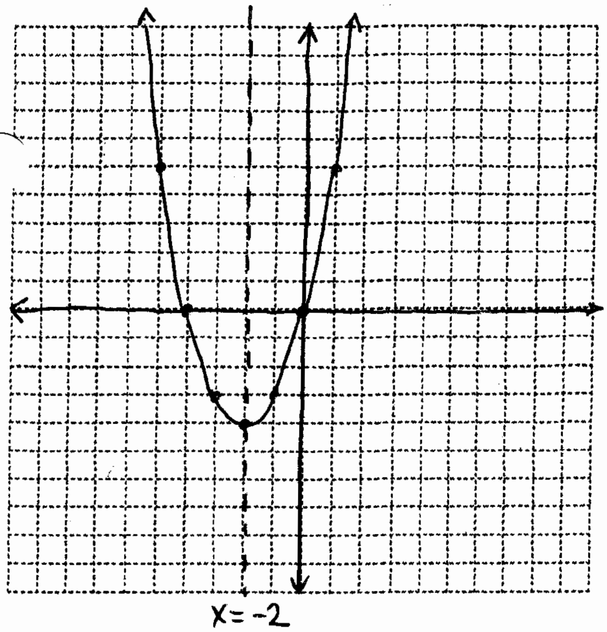
$\boxed{\text{ellipse}}$

d) $25x^2 + 25y^2 = 100$

x^2 and $y^2 \Rightarrow$ not a parabola

add $\Rightarrow$ not a hyperbola

coefficients $25x^2$ and $25y^2$ same $\Rightarrow \boxed{\text{circle}}$



① a) Graph $y = x^2 + 4x$ and its axis of symmetry

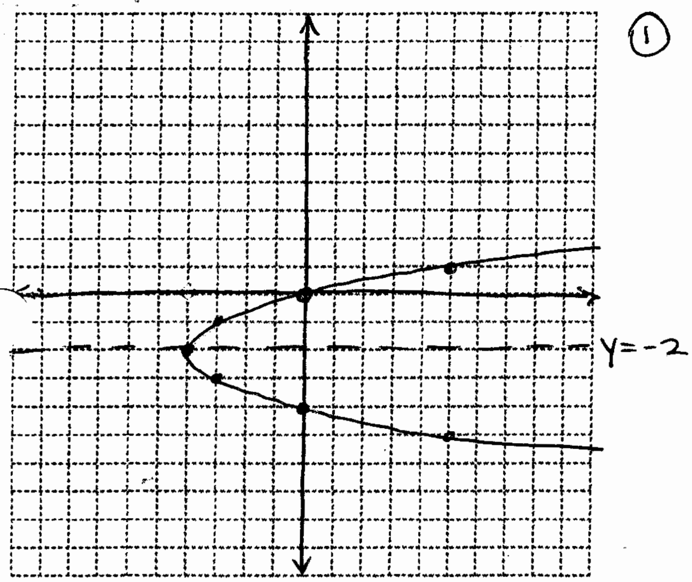
$y = (x+2)^2 - 4$

$a = 1 \Rightarrow$ standard shape

vertex $(-2, -4)$

up.

axis $x = -2$



① b) Graph $x = y^2 + 4y$ and its axis of symmetry

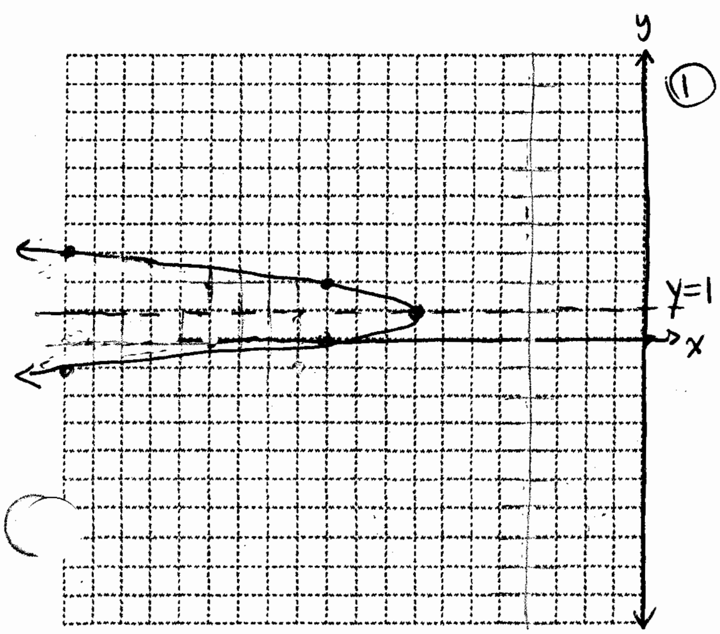
$x = (y+2)^2 - 4$

$a = 1 \Rightarrow$ standard shape

vertex $(-4, -2)$

right

axis $y = -2$



① c) Graph $x = -3y^2 + 6y - 11$

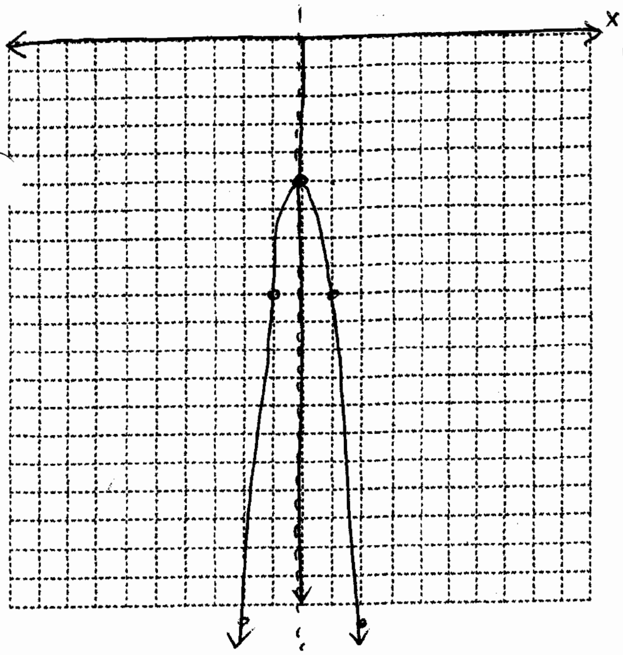
$x = -3(y-1)^2 - 8$

$a = -3 \Rightarrow$ narrow

vertex $(-8, 1)$

left

axis $y = 1$



1 d) $y = -4x^2 - 5$

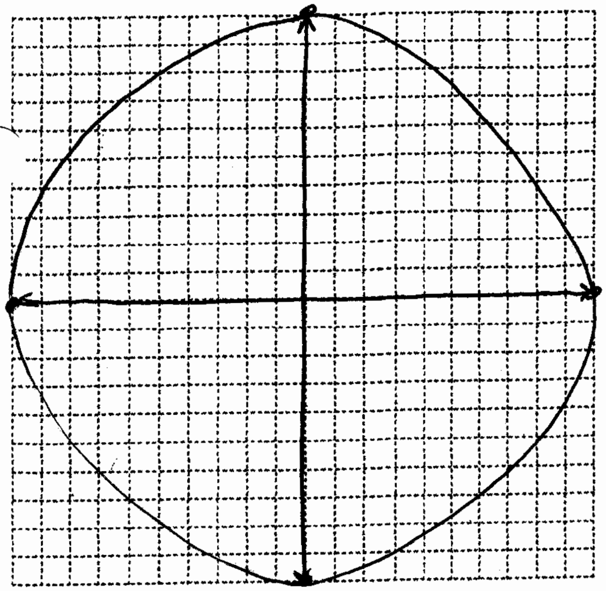
$y = -4(x-0)^2 - 5$

$a = -4 \Rightarrow$ narrow

vertex $(0,-5)$

down

axis $x=0$



③ a) $x^2 + y^2 = 100$

$x^2, y^2 \rightarrow$ not parabola

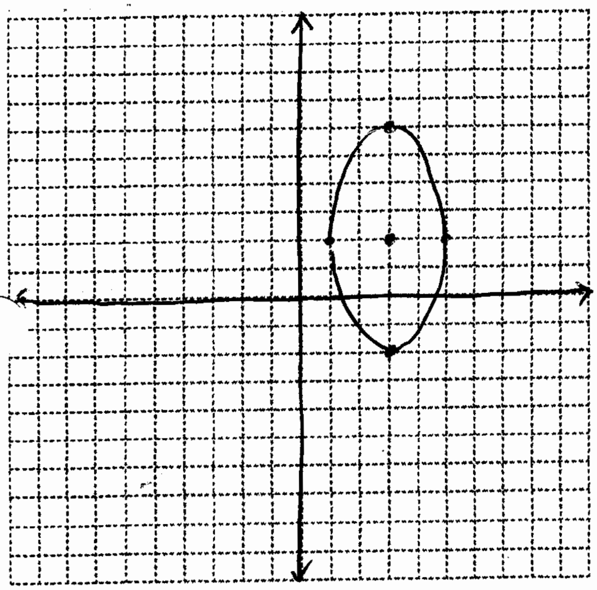
add $\rightarrow$ not hyperbola

same coef $\rightarrow$ circle

$(x-0)^2 + (y-0)^2 = 100$

center $(0,0)$

radius 10



③ b) $4(x-3)^2 + (y-2)^2 = 16$

x^2, y^2, added, diff coef $\Rightarrow$ ellipse

$*$ not $=1$ $\Rightarrow$ divide all by RHS$=16$

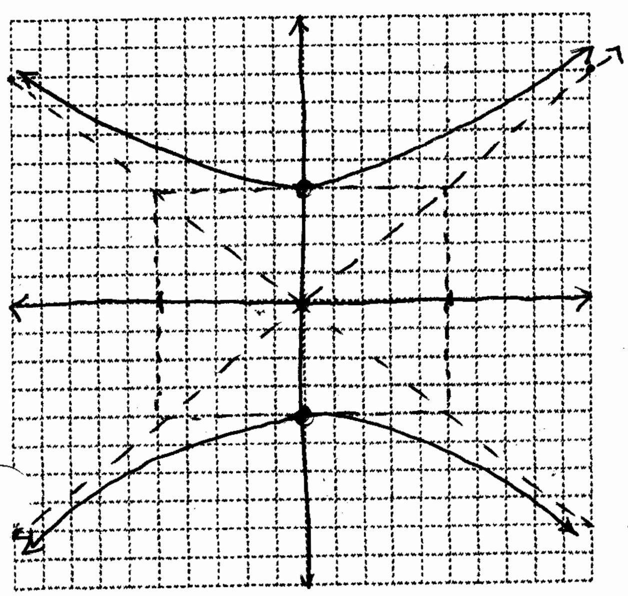
$$\frac{4}{16}(x-3)^2 + \frac{1}{16}(y-2)^2 = \frac{16}{16}$$

$$\frac{(x-3)^2}{4} + \frac{(y-2)^2}{16} = 1$$

center $(3,2)$

x-dir 2 $\rightarrow \sqrt{4}$ under x

y-dir 4 $\rightarrow \sqrt{16}$ under y



③ c) $\frac{y^2}{16} - \frac{x^2}{25} = 1$

x^2, y^2, subtracted $\Rightarrow$ hyperbola

$$\frac{(y-0)^2}{16} - \frac{(x-0)^2}{25} = 0$$

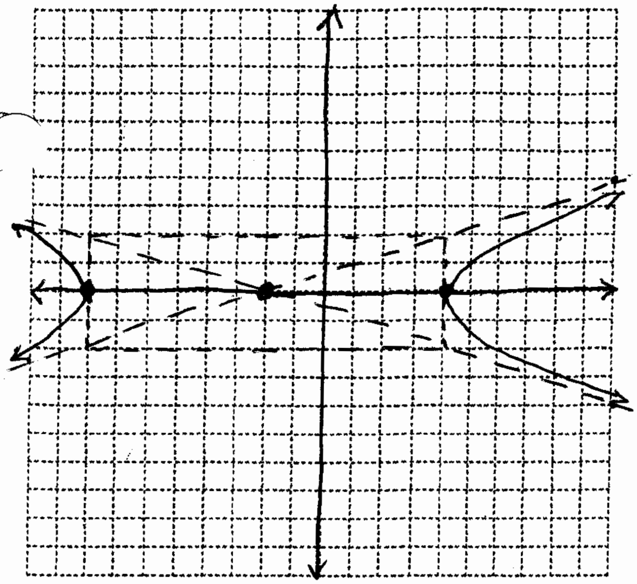
center $(0,0)$

x-dir $= 5 \rightarrow \sqrt{25}$ under x $\Big\}$ asymp-

y-dir $= 4 \rightarrow \sqrt{16}$ under y $\Big\}$ totes

y^2 first $\Rightarrow$ up/down hyperbola

plot $(0,4)$ and $(0,-4)$



3 d) $(x+2)^2 - 9y^2 = 36$

x^2, y^2, subtracted $\Rightarrow$ hyperbola

* not $= 1$ $\Rightarrow$ divide all by RHS $= 36$

$$\frac{1}{36}(x+2)^2 - \frac{9}{36}(y-0)^2 = \frac{36}{36}$$

$$\frac{(x+2)^2}{36} - \frac{(y-0)^2}{4} = 1$$

center $(-2,0)$

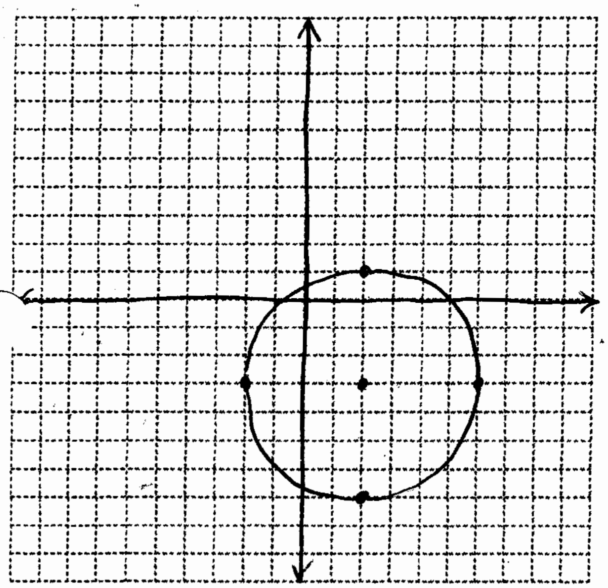
x-dir 6 $\rightarrow$ $\sqrt{36}$ under x

y-dir 2 $\rightarrow$ $\sqrt{4}$ under y

asymptotes

x^2 first $\Rightarrow$ left/right hyperbola

plot $(-8,0)$ and $(4,0)$



3 e) $x^2 + y^2 - 4x + 6y = 3$

x^2, y^2, added, same coef $\Rightarrow$ circle

* not in $(x-h)^2 + (y-k)^2 = r^2$ form $\Rightarrow$ CTS twice

$$(x^2 - 4x + 4) + (y^2 + 6y - \quad) = 3 + 4 + 9$$

$\frac{-4}{2} = -2$ $\quad (-2)^2 = 4$

$\frac{+6}{2} = 3$ $\quad 3^2 = 9$

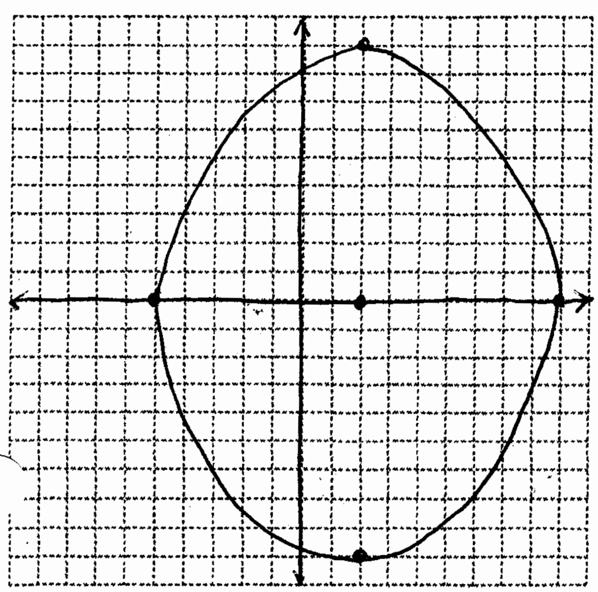
CTS x, CTS y

both sides

$$(x-2)^2 + (y+3)^2 = 16$$

center $(2,-3)$

radius 4



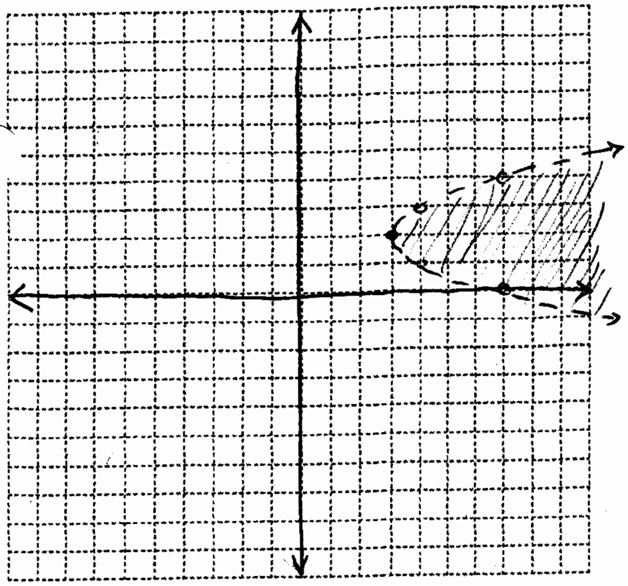
3 f) $\dfrac{(x-2)^2}{49} + \dfrac{y^2}{81} = 1$

x^2, y^2, added, diff coef $\Rightarrow$ ellipse

center $(2,0)$

x-dir 7 $\rightarrow$ $\sqrt{49}$ under x

y-dir 9 $\rightarrow$ $\sqrt{81}$ under y



④ a) $y^2 - 4y - x + 7 < 0$

$x, y^2 \Rightarrow$ parabola opening left or right
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad a<0 \quad\quad a>0$

$y^2 - 4y + 7 < x$

$x > y^2 - 4y + 7 \quad a = 1 \Rightarrow$ opens right

vertex formula $\quad y\text{ coord} = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$

$x\text{-coord} = (2)^2 - 4(2) + 7 = 3$

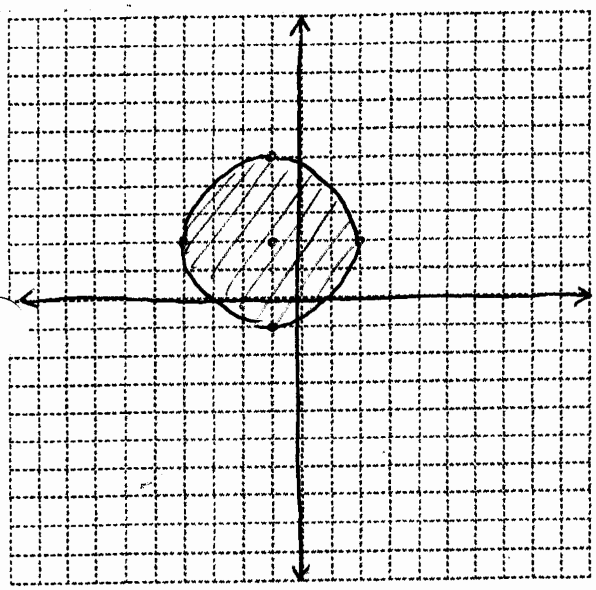
Vertex $(3,2)$ $\quad$ x-coord first!

$> \Rightarrow$ dashed boundary (no =)

$x>0$ shade right

OR Test $(0,0)$ $\quad 0 > 0^2 - 4(0) + 7$

$0 > 7$ false



④ b) $(x+1)^2 + (y-2)^2 \le 9$

x^2, y^2, added, same coef $\Rightarrow$ circle

center $(-1, 2)$

radius $3 = \sqrt{9}$

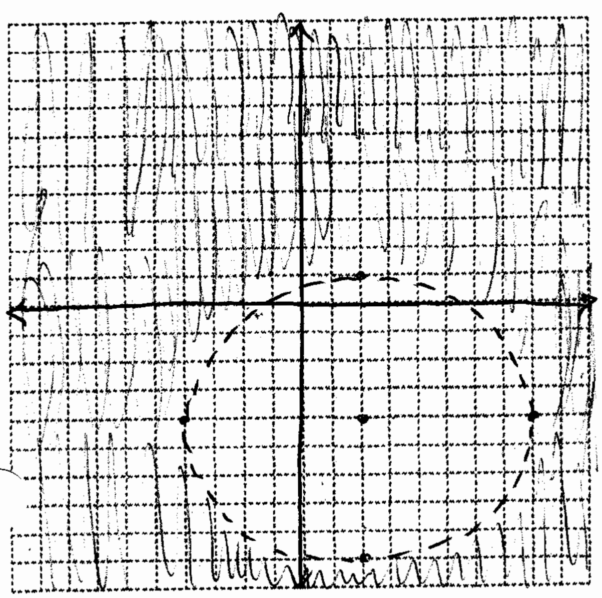
$\le$ solid boundary

$\le$ radius $\Rightarrow$ shade inside

OR Test center $(-1,2)$

$(-1+1)^2 + (2-2)^2 \le 9$

$0 \le 9$ true



④ c) $\frac{(x-2)^2}{36} + \frac{(y+4)^2}{25} > 1$

x^2, y^2, added, diff coef $\Rightarrow$ ellipse

center $(2, -4)$

x dir 6 $\longrightarrow \sqrt{36}$ under x

y dir 5 $\longrightarrow \sqrt{25}$ under y

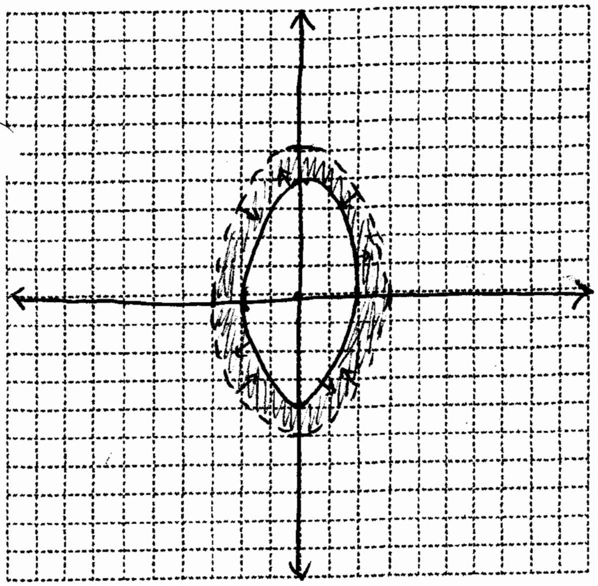
$>1 \Rightarrow$ dashed boundary (no =)

shade outside

OR Test center $(2,-4)$

$\frac{(2-2)^2}{36} + \frac{(-4+4)^2}{25} > 1$

$0 > 1$ false



⑤ a) $\begin{cases} 25x^2 + 9y^2 < 225 \\ 4x^2 + y^2 \geq 16 \end{cases}$

Both are x^2, y^2, added, diff coef $\Rightarrow$ ellipse

$\frac{25x^2}{225} + \frac{9y^2}{225} < \frac{225}{225}$

dashed boundary
inward

$\frac{x^2}{9} + \frac{y^2}{25} < 1$

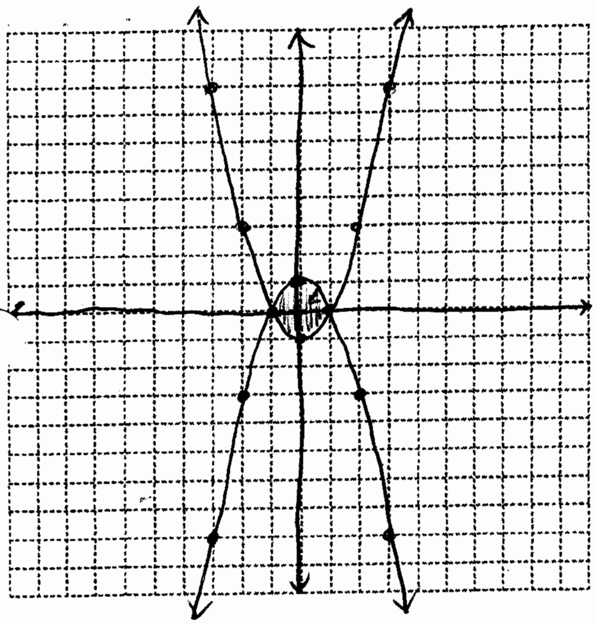
center $(0,0)$ xdir=3
ydir=5

$\frac{4x^2}{16} + \frac{y^2}{16} \geq \frac{16}{16}$

solid boundary
outward

$\frac{x^2}{4} + \frac{y^2}{16} \geq 1$

center $(0,0)$ xdir=2
ydir=4



⑤ b) $\begin{cases} x^2 \leq 1 - y \\ x^2 \leq y + 1 \end{cases}$

Both are x^2, y
$\Rightarrow$ parabolas up or down

$x^2 \leq 1 - y$
$y \leq -x^2 + 1$
$y \leq -(x-0)^2 + 1$

a=-1 opens down
vertex $(0,1)$
solid boundary
shade down

$x^2 \leq y + 1$
$y + 1 \geq x^2$
$y \geq x^2 - 1$
$y \geq (x-0)^2 - 1$

a=1 opens up
vertex $(0,-1)$
solid boundary
shade up

⑥ Write equation of circle

a) center at origin, radius 1

$(0,0) \qquad r=1$

$$(x-0)^2+(y-0)^2=1^2$$

$$\boxed{x^2+y^2=1}$$

b) center at origin, radius $\sqrt{7}$

$$(x-0)^2+(y-0)^2=(\sqrt{7})^2$$

$$\boxed{x^2+y^2=7}$$

c) center at $(-2,1)$, radius 3

$$(x-(-2))^2+(y-1)^2=3^2$$

$$\boxed{(x+2)^2+(y-1)^2=9}$$

d) center $(3,-7)$, radius 11

$$(x-3)^2+(y-(-7))^2=11^2$$

$$\boxed{(x-3)^2+(y+7)^2=121}$$

⑦ Solve, write ordered pair

a) $\begin{cases} y=3x-5 & Ⓐ \\ x^2+y^2=5 & Ⓑ \end{cases}$

substitution method because y isolated in Ⓐ, subst Ⓐ into Ⓑ

$x^2+(3x-5)^2=5$ — subst for y, use ( ).

$x^2+(3x-5)(3x-5)=5$ — exponent means FOIL

$x^2+9x^2-15x-15x+25=5$

$10x^2-30x+25=5$ — combine like terms

see $x^2 \Rightarrow$ quadratic equation — set $=0$

$10x^2-30x+20=0$

$10(x^2-3x+2)=0$

$10(x-2)(x-1)=0$

$$\begin{array}{ccc} & 2 & \\ -2 & \times & -1 \\ & -3 & \end{array}$$

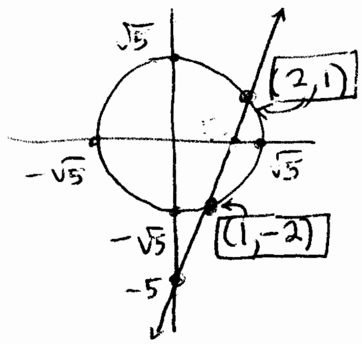
$x=2$ or $x=1$

$y=3(2)-5 \qquad y=3(1)-5$

$=6-5 \qquad\quad =3-5$

$$\boxed{(2,1) \qquad\qquad (1,-2)}$$

check by graphing intersections ... OR ...



check by algebra

$$\left.\begin{aligned}1&=3(2)-5\\2^2+1^2&=5\end{aligned}\right\}\checkmark$$

(2,1) works in both equations

$$\left.\begin{aligned}-2&=3(1)-5\\1^2+(-2)^2&=5\end{aligned}\right\}\checkmark$$

(1,-2) works in both equations

b) $\begin{cases} x^2 - 3y^2 = -1 & (A) \\ 2x^2 - 7y^2 = -5 & (B) \end{cases}$

elimination method

★ option 1: to eliminate x^2, mult (A) by -2.

option 2: to eliminate y^2, mult (A) by 7 and (B) by -3 yuck.

$-2x^2 - 3(-2)y^2 = -1(-2)$ (A) $\times(-2)$ on both sides

$\begin{cases} -2x^2 + 6y^2 = 2 \\ 2x^2 - 7y^2 = -5 \end{cases}$ simplify (A); (B)

$-y^2 = -3$ add like terms on both sides

$y^2 = 3$ mult (or divide) by -1 to isolate y^2

$y = \pm\sqrt{3}$ square root property

subst back to find x coordinates.

$y = \sqrt{3} \Rightarrow$ (A)

$x^2 - 3(\sqrt{3})^2 = -1$

$x^2 - 3 \cdot 3 = -1$

$x^2 - 9 = -1$

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

$y = -\sqrt{3} \Rightarrow$ (A)

$x^2 - 3(-\sqrt{3})^2 = -1$

$x^2 - 3(+3) = -1$

exp before mult.

$x^2 - 9 = -1$

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

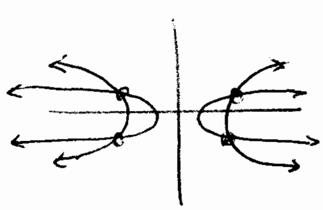
$\Rightarrow$ $\boxed{(2\sqrt{2}, \sqrt{3}) \text{ and } (-2\sqrt{2}, \sqrt{3})}$ $\boxed{\text{and}}$ $\boxed{(2\sqrt{2}, -\sqrt{3}) \text{ and } (-2\sqrt{2}, -\sqrt{3})}$

check by algebra

$(2\sqrt{2})^2 - 3(\sqrt{3})^2 = 4 \cdot 2 - 3 \cdot 3 = -1$ ✓ negatives don't matter

$2(2\sqrt{2})^2 - 7(\sqrt{3})^2 = 2 \cdot 4 \cdot 2 - 7 \cdot 3 = 16 - 21 = -5$ ✓

two hyperbolas centered at $(0,0)$



yes, four solutions!

c) $\begin{cases} y = x^2 - 1 & \text{(A)} \\ y = -4x - 5 & \text{(B)} \end{cases}$

option 1: eliminate y
option 2: substitution for y } Both approaches yield same result...

option 1:

$$\begin{array}{rll} y = & x^2 - 1 & \text{(A)} \\ -y = & 4x + 5 & \text{(B)} \times (-1) \\ \hline 0 = & x^2 + 4x + 4 & \end{array}$$

$(x+2)^2 = 0$

$x = -2$

option 2:

$x^2 - 1 = -4x - 5$

$x^2 + 4x + 4 = 0$

Quadratic equation
factor

use (B) lower degree to find y-coordinate

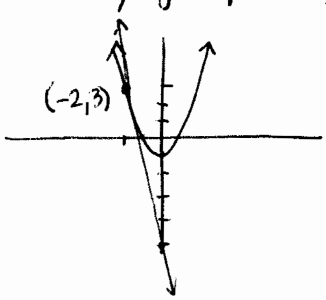
$y = -4(-2) - 5$

$y = 8 - 5$

$y = 3$

$\boxed{(-2, 3)}$

check by graphing



⑧ Find the first 5 terms and 100th term

a) $a_n = 4 - n$

$n=1 \quad a_1 = 4-1 = 3$
$n=2 \quad a_2 = 4-2 = 2$
$n=3 \quad a_3 = 4-3 = 1$
$n=4 \quad a_4 = 4-4 = 0$
$n=5 \quad a_5 = 4-5 = -1$

$\boxed{3, 2, 1, 0, -1}$

$n=100 \quad a_{100} = 4-100 = \boxed{-96}$

b) $b_n = 2^{n-1}$

$n=1 \quad b_1 = 2^{1-1} = 2^0 = 1$
$n=2 \quad b_2 = 2^{2-1} = 2^1 = 2$
$n=3 \quad b_3 = 2^{3-1} = 2^2 = 4$
$n=4 \quad b_4 = 2^{4-1} = 2^3 = 8$
$n=5 \quad b_5 = 2^{5-1} = 2^4 = 16$

$\boxed{1, 2, 4, 8, 16}$

$n=100 \quad b_{100} = 2^{100-1} = \boxed{2^{99}}$ (has 30 digits left of decimal point)

c) $c_n = \dfrac{n^2}{n+1}$

$n=1 \quad c_1 = \dfrac{1^2}{1+1} = \dfrac{1}{2}$

$n=2 \quad c_2 = \dfrac{2^2}{2+1} = \dfrac{4}{3}$

$n=3 \quad c_3 = \dfrac{3^2}{3+1} = \dfrac{9}{4}$

$n=4 \quad c_4 = \dfrac{4^2}{4+1} = \dfrac{16}{5}$

$n=5 \quad c_5 = \dfrac{5^2}{5+1} = \dfrac{25}{6}$

$\boxed{\dfrac{1}{2}, \dfrac{4}{3}, \dfrac{9}{4}, \dfrac{16}{5}, \dfrac{25}{6}}$

$n=100 \quad c_{100} = \dfrac{100^2}{100+1} = \boxed{\dfrac{10000}{101}}$

⑨ Find the general term.

a) 1,2,3,4

$\boxed{a_n = n}$

or...
add one each time ⇒ arithmetic sequence

$a_n = a_1 + d(n-1)$

$a_n = 1 + 1(n-1)$

$a_n = 1 + n - 1$

$\boxed{a_n = n}$

b) 9, 6, 3, 0, ...

subtract 3 each time ⇒ arithmetic sequence with add (-3)

$a_1 = 9$

$d = -3$

$a_n = a_1 + d(n-1)$

$= 9 + (-3)(n-1)$

$= 9 - 3n + 3$

$\boxed{a_n = 12 - 3n}$

c) 8, 4, 2, 1, ...

divide by 2 each time ⇒ geometric sequence with mutiply by $\frac{1}{2}$

$a_1 = 8$

$r = \frac{1}{2}$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 8 \cdot \left(\frac{1}{2}\right)^{n-1}}$

$8 = 2^3 = \left(\frac{1}{2}\right)^{-3}$

$= \left(\frac{1}{2}\right)^{-3}\left(\frac{1}{2}\right)^{n-1}$

$= \left(\frac{1}{2}\right)^{-3+n-1}$

$\boxed{a_n = \left(\frac{1}{2}\right)^{n-4}}$ or $\boxed{a_n = \frac{1}{2^{n-4}}}$

d) 2, 6, 18, 54

mult by 3 each time ⇒ geometric sequence

$a_1 = 2$

$r = 3$

$a_n = a_1 r^{n-1}$

$\boxed{a_n = 2 \cdot 3^{n-1}}$

e) $5, 11, 21, 35, 53, \ldots$

$+6 \quad +10 \quad +14 \quad +18$

add an increasing # each time $\Rightarrow n^2$'s

$\Rightarrow a_n = xn^2 + yn + z$

$n=1 \quad 5 = x(1)^2 + y(1) + z \quad \Rightarrow \quad 5 = x + y + z$

$n=2 \quad 11 = x(2)^2 + y(2) + z \quad \Rightarrow \quad 11 = 4x + 2y + z$

$n=3 \quad 21 = x(3)^2 + y(3) + z \quad \Rightarrow \quad 21 = 9x + 3y + z$

3X3 system

$x = 2$

$y = 0$

$z = 3$

$a_n = 2n^2 + 0n + 3$

$\boxed{a_n = 2n^2 + 3}$

(10) Use summation notation

a) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$

$\uparrow \quad n=1 \qquad \uparrow \quad n=5$

$a_n = \frac{n}{n+1}$

$$\boxed{\sum_{n=1}^{5} \frac{n}{n+1}}$$

b) $2 + 7 + 12 + 17 + 22$

+5 each time $\Rightarrow$ arithmetic sequence

$a_1 = 2$

$d = 5$

$a_n = a_1 + d(n-1)$

$a_n = 2 + 5(n-1)$

$= 2 + 5n - 5$

$= 5n - 3$

$$\boxed{\sum_{n=1}^{5} (5n-3)}$$

c) $\frac{1}{4} + \frac{2}{5} + \frac{1}{2} + \frac{4}{7} + \frac{5}{8}$

$\Uparrow$

$\frac{1}{2} = \frac{3}{6}$

counting from 1

$\frac{1}{4} + \frac{2}{5} + \frac{3}{6} + \frac{4}{7} + \frac{5}{8}$ $\leftarrow$ counting from 4

$a_n = \frac{n}{n+3}$

$$\boxed{\sum_{n=1}^{5} \frac{n}{n+3}}$$

(11) Evaluate the sum

a) $\displaystyle\sum_{i=1}^{9} 2^i$

$= 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9$

$= 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512$

$= \boxed{1022}$

b) $\displaystyle\sum_{i=5}^{11} \frac{1}{i+3}$

$$= \frac{1}{5+3} + \frac{1}{6+3} + \frac{1}{7+3} + \frac{1}{8+3} + \frac{1}{9+3} + \frac{1}{10+3} + \frac{1}{11+3}$$

$$= \underbrace{\frac{1}{8} + \frac{1}{9}} + \underbrace{\frac{1}{10} + \frac{1}{11}} + \underbrace{\frac{1}{12} + \frac{1}{13}} + \frac{1}{14}$$

$\approx .6587051837$ (my Gc. Math $\triangleright$ frac)

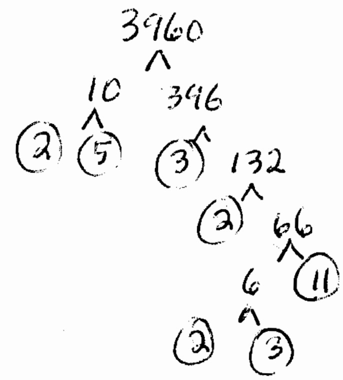
$$= \underbrace{\frac{17}{72} + \frac{21}{110}} + \underbrace{\frac{25}{156} + \frac{1}{14}}$$

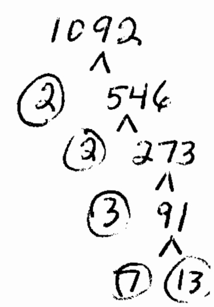
$$= \frac{1691}{3960} + \frac{253}{1092}$$

$$= \frac{1691}{3960} \cdot \frac{7}{7} \cdot \frac{13}{13} + \frac{253}{1092} \cdot \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{5}{5} \cdot \frac{11}{11}$$

$$= \frac{153881 + 83490}{360360}$$

$$= \boxed{\frac{237371}{360360}} \approx .6587051837$$





$2^3 \cdot 3^2 \cdot 5 \cdot 11$ $\qquad$ $2^2 \cdot 3 \cdot 7 \cdot 13$

$\text{LCD} = 2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$

$= 360360$